AIAA Journal

VOLUME 3 MARCH 1965 NUMBER 3

Physics of Meteor Entry

MARY F. ROMIG
The Rand Corporation, Santa Monica, Calif.

Nomenclature*

A = cross-sectional area
A^* = reference area, Eq. (20)
C_D = drag coefficient
C_H = heat-transfer coefficient
E = integrated zero-magnitude intensity
E_k = kinetic energy
F_d = drag force
h = altitude
H = enthalpy
I = zero-magnitude light intensity
K = density parameter, Eq. (19)
$\mathfrak{M}$ = magnitude
m = mass
P = power
q = heat-transfer rate
Q = effective heat of ablation or vaporization
R = radius
Re = Reynolds number
t = time
T = temperature, °K
v = velocity
α = inverse scale height
γ = angle between flight path and horizontal
ϵ = function denoting second-order effects, Eq. (4)
λ = mean free path
ρ = density
σ = entry parameter, Eq. (23)
τ = luminous efficiency factor
τ_0 = luminosity coefficient
ϕ = ballistic parameter = $2m\alpha \sin\gamma / C_D A$

Subscripts

B = body
BL = boundary layer
e = end of luminous flight
p = photometric
r = reflected
s = stagnation
SL = sea level
∞ = conditions at entry

I. Introduction

METEORS have always been of casual interest to the aerodynamicist, since the typical meteor is often used as a classic example of hypervelocity flight in free-molecule flow. Unfortunately, the example has become stereotyped to the extent that even very bright meteors, or fireballs, which penetrate deeply into the atmosphere are sometimes assumed to experience free-molecule flow. Recently, however, some aerodynamicists have turned toward the field of meteor entry as a means of deducing information about forthcoming problems associated with re-entry of superorbital missions. It is quite likely that these studies will also contribute much toward filling the present breach between the fields of meteor astronomy and aerodynamics.

Although a natural laboratory such as this is relatively inexpensive and abundant in subjects, it has shortcomings. The experiment is obviously uncontrolled. The ambient conditions are not well defined (compared to flight in ballistic ranges, for example). The mass and the shape of the object are not known, nor is the density of the meteoric material (unless an associated meteorite is found). The velocity and deceleration can be accurately deduced only if the flight is photographed or observed with radar. The velocities encountered are, at their lowest, higher than those associated with near-space re-entry. Meteors do provide, however, a glimpse of flight under conditions that cannot easily be duplicated in terrestrial laboratories.

This paper briefly surveys the present status of the physics of meteor entry. Since meteors range in size from very small to very large, and in velocities from 11 to 72 km/sec, it will be necessary at the outset to define the flow regimes and to illustrate the importance, as far as the kinetic energy deposition is concerned, of assuming the correct flow regime for solution of the dynamical equations. The primary objective in such a solution is to determine the mass and density of the object, and the various methods presently in use will be discussed. The effect of various assumptions (primarily those concerning heat transfer) on the mass loss will be shown for the entry of

Miss Romig is a physical scientist at The Rand Corporation. She has been active in the fields of heat transfer and high-speed gasdynamics since 1951 and was an Associate Fellow of the Institute of Aeronautical Sciences. She is a graduate of Vassar College and has done graduate work in mathematics and engineering at Stanford and UCLA. Her research interests have been concentrated recently in the areas of plasmadynamics and hypervelocity entry, in particular at speeds exceeding 11 km/sec. As an Associate Fellow Member of the AIAA, she is on the AIAA Plasmadynamics Committee.

Received May 12, 1964; revision received November 9, 1964. Any views expressed in this paper are those of the author. They should not be interpreted as reflecting the views of The Rand Corporation or the official opinion or policy of any of its governmental or private research sponsors. Papers are reproduced by The Rand Corporation as a courtesy to members of its staff.

* Nomenclature is in cgs units.

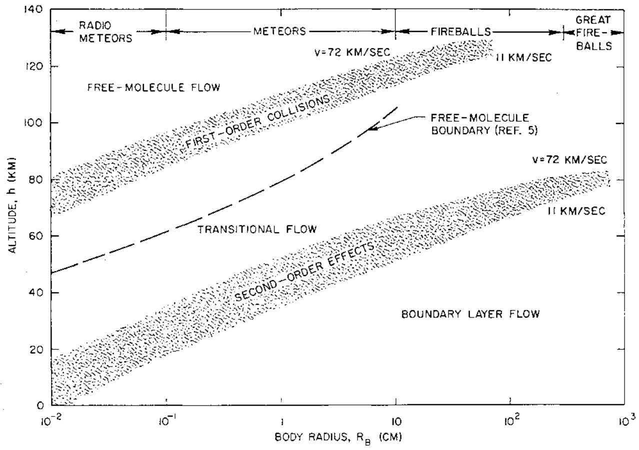

Fig. 1 Flow regimes encountered during the hypervelocity entry of various bodies.

the Přibram and Meanook meteors. Radar techniques are not discussed.

II. Flow Regimes

If the deposition of kinetic energy by the meteoroid into various "observable" forms were known, the task of the meteor physicist would be relatively simple. It would then be possible, for example, to measure the radiated light and velocity and deduce the mass, if it were known how much of the kinetic energy were deposited in the form of light. This approach is, in fact, one which is used quite commonly. However, the theory is based on concepts associated with free-molecule flow; the relative kinetic energy of the intercepted air particles is transferred to the body in the form of heat, and the material ejected from the surface produces light and ion pairs upon collision with ambient air particles.[1] It is estimated[2] that the kinetic energy lost by the body is deposited in the form of heat:light:ionization in the ratio 10^4:10^2:10. The total power dissipation for a Perseid, for example, is on the order of 100 Mw. This theory predicts that very little of the energy is radiated in the form of light, and even less is given up to ionization.

To illustrate the partition of kinetic energy loss, one can write a power balance

$$F_d v \equiv \frac{dE_k}{dt} \equiv \frac{1}{2} v^2 \frac{dm}{dt} + mv\frac{dv}{dt} \equiv -(P_B + P_R + P_A) \quad (1)$$

The sink term P_B represents the heat transferred to the body and contains the latent heat of the surface material. The radiative term P_R contains energy radiated (over the entire spectrum) from the gas, surface, and ablated material. The last term is the energy dissipated into the ambient air and accounts for the production of ions and electrons.

As a body of given size penetrates into the atmosphere it will encounter three general flow regimes as shown in Fig. 1. As a convenience for later discussions, the pertinent dimension is taken to be the radius of a sphere, although the criteria of Fig. 1 apply equally well to arbitrary shapes with typical dimension R_B. The boundaries indicated in Fig. 1 are tentative and for that reason are indicated by shaded regions. The mean free path variation used in preparing Fig. 1 was obtained from Ref. 3.

The highest altitude regime is the free-molecule regime and is generally denoted by the inequality $\lambda/R_B \gg 1$. Under these circumstances, ambient air particles in the region of the body collide with the body rather than with themselves, so that there is maximum energy transfer from the freestream to the surface. As the ambient density increases, or as the body begins to vaporize (or sputter), the particles leaving the surface will begin to interact with the incoming air.

First-order collisions between reflected and incoming air particles shield the body from direct impact and lower the term P_B in Eq. (1). Specifying that the local Knudsen number associated with the reflected particle is[4]

$$\lambda_r/R_B = \lambda v_r/2^{1/2}R_B v > 1 \quad (2)$$

we can obtain approximate lower bounds to the free-molecule regime. This is shown in Fig. 1 for diffuse reflection from a cool wall ($v_r \ll v$) for $11 \leq v \leq 72$ km/sec. Unfortunately, little is known about the reflection of air particles from iron and stone when the impact energy ranges from 10^2 to 10^3 ev. If the reflection were more specular, this boundary would move to lower altitudes for a given velocity and radius. Levin[5] states that the reflection from stone and iron meteoroids is different and that the air molecules rebound from iron with a larger velocity than the thermal velocity. Therefore, one could neglect shielding effects for iron meteoroids to much lower altitudes (those indicated by the dashed line in the transition zone). Levin's curve for stone meteoroids agrees roughly with the upper boundary of Fig. 1.

The transitional regime contains those flows which are difficult to treat analytically.[6] Here the shock wave begins to form and viscous and conductive effects in the shock layer are important. Boundary-layer flow, in which the usual continuum theory applies and the shock is nominally a discontinuity, is approached through the zone marked "second-order effects" in Fig. 1. According to Van Dyke,[7] the stagnation-point heat transfer can be written in terms of the continuum theory as

$$\frac{q}{q_{BL}} = 1 + \{\text{vorticity} + \text{curvature} + \text{slip} + \text{temperature jump}\}\epsilon \quad (3)$$

where the terms within brackets denote second-order corrections to the boundary-layer equations and are on the order of 1. When

$$\epsilon = (1/Re_s)^{1/2} \leq 0.01 \quad (4)$$

there is less than 1% difference in the heat transfer. The lower shaded zone in Fig. 1 was obtained from normal shock calculations for air in thermodynamic equilibrium[8] and a viscosity which varied as $T_s^{1/2}$. Since radiative effects on the shock structure were also neglected, the extent of this zone is highly qualitative. If, for example, the air were not in equilibrium, the temperature T_s would be higher, and second-order effects would occur at lower altitudes for the same radius and velocity. On the other hand, if the effect of radiation from the shock layer were to decrease the temperature T_s (and increase the freestream temperature), the curves would move up into the present transition zone.

During the progression from free-molecule to continuum flow, the heat-transfer term P_B in Eq. (1) will gradually decrease, and the last term P_A will increase as the shock wave becomes more well defined. A large portion of the energy radiated from the shock at these velocities will be in the ultraviolet, producing O_2^+ and electrons in front of the shock by photoionization. The associated halo is believed to be the cause of radar "head echoes"[9] as distinct from reflection from the ionized trail. The radiative term P_R should increase because of the contributions from shock-heated air and recombination in the wake. In this connection, it is interesting to note that most of the meteor spectra discussed by Kramer et al.[10] show a strong unresolved system due to the nitrogen first positive group. This radiation was also seen during the entry of the Meanook fireball[11] and a fireball of -9 magnitude† in

† One zero magnitude intensity is equivalent to 2.73×10^{-11} kw/m² at 100-km alt. In general, $\mathfrak{M} = -2.5 \log_{10} I$, where $\mathfrak{M}$ is the photographic magnitude and I is the light intensity in units of zero magnitude.

the Soviet Union.[12] In the latter, the usual lines due to ionized elements were absent, and the radiation was due only to metallic atoms and the molecular nitrogen. These nitrogen bands have been seen previously only in connection with the faster Perseids[13] at much higher altitudes. Perseids have also exhibited the O*I* line at 7774 Å and several multiplets of N*I*.[14, 15] The oxygen may be due to stony meteoric material, but the nitrogen is almost certainly due to air radiation (perhaps from recombination radiation in the wake).

As yet there is no analytic technique of predicting the relative magnitude of these power terms for the lower flow regimes; it is only in the free-molecule regime that the ratios are established. It is of importance, therefore, to determine which flow regime is valid when the entry of a meteor is analyzed. Figure 2 shows the altitudes of luminous flight for meteors of various brightness. (These are denoted in Fig. 1 as a function of size from the designation established by Whipple and Hawkins.[16]) Over 300 photographic meteors, of brightness ranging from magnitude of -3 to $+3$, were statistically analyzed by Hawkins and Southworth.[17] It is seen from Fig. 1 that these objects experience luminous flight in near free-molecule flow if they are less than 1 cm in radius. At the other extreme, the average luminous flight regime for fireballs (magnitude -3 to -20) encompasses the entire gamut of flows. (These data were obtained from Olivier.[18]) Two trajectories of photographed fireball entires[11, 19] are also shown on Fig. 2. These will be discussed in some detail later.

III. Relevant Equations

The dynamical equations are usually written in terms of the aerodynamic coefficients as

$$dv/dt = -(C_D A/2m)\rho v^2 \tag{5}$$

$$dm/dt = -(C_H A/2Q)\rho v^3 \tag{6}$$

where gravitational effects are neglected. The heat-transfer coefficient C_H can include contributions from convection and radiation. Usually, the density variation is assumed to follow an exponential law

$$\rho = \rho_{SL} e^{-\alpha h} \tag{7}$$

where $1/\alpha$ is a scale height that can be variable.

When the ballistic parameter $\phi = (2m\alpha/\sin\gamma)/(C_D A)$ is assumed constant, Eq. (5) can be solved and the altitude of peak deceleration can be found. The general practice is to let the entry angle, drag coefficient, and scale height be constant. All of these are valid assumptions at altitudes from about 100 km to that altitude where the body begins to decelerate appreciably. Therefore, a constant ϕ during entry implies that m/A is constant as the body melts. Equivalently, any change in a characteristic dimension (such as the radius for a sphere) must be small in order for the following solution to be valid.

Integration of Eq. (5), with the use of Eq. (7) and the relationship $dh/dt = -v \sin \gamma$, yields

$$v/v_\infty = \exp(-\rho/\phi) \tag{8}$$

The altitude of peak deceleration is reached when $v/v_\infty = 0.607$.

If the heat-transfer coefficient is constant, maximum heating occurs when

$$[(\rho/v)(dv/d\rho)] = -\tfrac{1}{3} \text{ or when } \rho = \phi/3 \tag{9}$$

Fay, Moffatt, and Probstein[20] show that if the heat-transfer coefficient is written‡

$$C_H \sim \rho^{-n} v^{-m} \tag{10}$$

‡ In order to be strictly correct, Eq. (10) should include a characteristic body dimension to some power; however, in accordance with the assumption that m/A = const, the variation in body dimension is neglected in deriving Eq. (11).

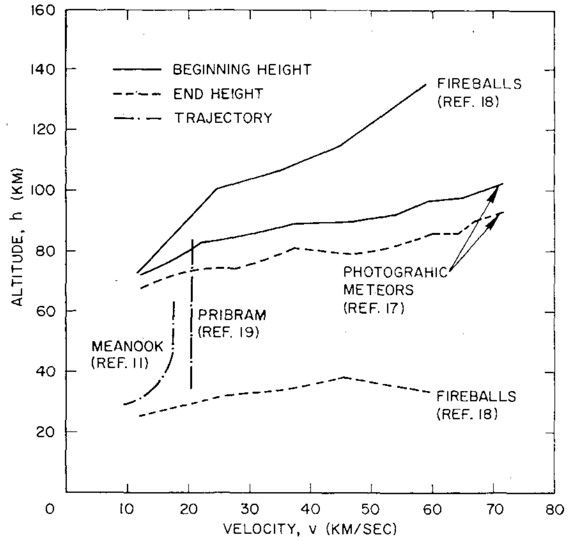

Fig. 2 Altitudes of luminous flight for telescopic meteors and fireballs. Partial trajectories of two photographed fireballs also are shown.

then the altitude of peak heating is determined by

$$\rho = \phi[(1-n)/(3-m)] \tag{11}$$

If the density at maximum heating, given by Eq. (11), is less than 2ϕ, peak heating occurs at higher altitudes than peak deceleration.

In addition to Eqs. (5) and (6), an empirical relationship is often used. This is based on the free-molecule concept that a known portion of the kinetic energy dissipation is radiated in the form of light [see, e.g., Eq. (1)]. Neglecting the deceleration in Eq. (1), we can write

$$dE_k/dt \sim \tfrac{1}{2}v^2 (dm/dt) \tag{12}$$

According to Verniani,[21] the photographed spectra of meteors show that the kinetic energy lost by deceleration does not contribute to the light. Then the luminosity equation is written

$$I = -(\tau/2)v^2(dm/dt) \tag{13}$$

where τ is the luminous efficiency factor. That the light intensity should be proportional to the ablation rate is clear. The dependence of I on the velocity is less straightforward; essentially, the spectral distribution of the light will vary with the velocity. The relationship, however, will probably be quite complicated. It is generally agreed to express a part of this dependence by letting τ vary as some power of the velocity. Most evidence indicates that

$$\tau = \tau_0 v \tag{14}$$

so that

$$dm/dt = -(2/\tau_0)(I/v^3) \tag{15}$$

provides an additional equation for the mass determination. The factor τ_0 is called a luminosity coefficient.§

The gas cap radiation was neglected in this discussion of luminosity. According to Verniani,[21] this neglect is justified since the only experimental evidence of such radiation indicates it is important only in the red and infrared regions,[11] whereas the meteors used for the deduction of τ_0 were photographed in blue emulsion. Although it is true that only the N_2 first positive bands have been seen on meteor spectra, a great deal of gas cap radiation is known to occur in the blue and ultraviolet,[22] which may not be detectable on the par-

§ The units of τ_0 are zero magnitude sec^4/g-cm^3, where I is the luminosity in units of zero magnitude.

Table 1 Estimates of the luminosity coefficient

Source	Restrictions	τ_0
Jacchia[24]:	$v > 20$ km/sec	6.47×10^{-19}
Cook et al.[27]:	$R_B < 1$ cm	
	Stone	4.26×10^{-19}
	Iron	2.76×10^{-19}
McCrosky and Soberman[25]:	$R_B \leq 2$ cm	
	$v = 10$ km/sec	
	Iron (93% Fe)	7.4×10^{-19}
	Stone (15% Fe)	1.2×10^{-19}
	Cometary (1% Fe)	8.0×10^{-21}
	(20% Fe)	1.6×10^{-19}
Verniani[21]:	...	1.0×10^{-19}

ticular instruments used for observing meteors. Such gas cap radiation is of importance in determination of the heat transfer as well as the production of light. It is also entirely possible that the nitrogen bands reported in Refs. 11 and 12 were due to recombination radiation in the wake rather than collisional excitation in the shock.[23]

Even though any gas cap radiation occurring at the lower wavelengths will not be observable on spectral plates, it still participates in the energy balance of Eq. (1). Furthermore, this radiation is a function of air density (if the air is in equilibrium) as well as velocity. Other factors affect the meteor luminosity, in particular, the composition and shape. It is difficult to expect Eq. (15), or equivalently, a single value of τ_0, to be universally applicable to the entry of all types of meteors.

Some of these shortcomings in this empirical approach have been recognized by meteor physicists. The uncertainty in τ_0 (due primarily to composition and fragmentation) can be as large as two orders of magnitude.[16] A critical summary of recent work on the functional form of τ may be found in Ref. 21. Several estimates of τ_0 are given in Table 1. The standard value deduced by Jacchia[24] is also quoted. McCrosky and Soberman[25] measured a value for τ_0 at 10 km/sec using results from a Trailblazer rocket carrying a small iron pellet. Assuming that all the luminosity in the range 3500–5000 Å was produced by iron line emissions, they obtained lower limits for τ_0 which are also given in Table 1.

Equations (5, 6, and 15), along with a suitable density variation, provide the framework for the determination of mass. In general, the velocity, deceleration, path angle, and light intensity are known as functions of altitude (or time). The unknowns, besides the mass, are the aerodynamic coefficients, heat of vaporization or melting (a function of body material), and cross-sectional area. It will be generally assumed in the following discussion that the drag coefficient is constant, either assuming the free-molecule value of $C_D = 2$ or the Newtonian value of $C_D = 1$. This assumption does not take into account the "reverse jet effect" of vaporizing molecules, whose momentum tends to increase C_D above the free-molecule value.[26]

IV. Methods of Solution

A survey of the literature shows that several techniques are presently used to determine meteoric mass and density (see, e.g., Ref. 5, the summaries in Astapovich,[28] Bronshten,[29] or Whipple and Hawkins[16]). The problem of primary importance is to determine a mass-loss law as a function of velocity, altitude (or density), and body size. To do this, one can use empirical results [e.g., Eq. (15)], extrapolate presently known formulas for heat transfer, or derive new equations. The first of these is employed extensively in reducing meteor data and is generally referred to as the photometric approach.

Table 2 Values of density parameter K

Source	$\log_{10} K$ cgs	Comments
Jacchia[24]:	6.403	$\log \tau_0 = -18.19$
Whipple[30]:	6.321	$\log \tau_0 = -19.42$ (stone cube, $C_D = 1.22$)
Cook et al.[27]:	6.61	$\log \tau_0 = -18.71$ (stone sphere, $C_D = 2$)
	6.91	sphere, $C_D = 1$
	6.29	cometary composition (low density)

Photometric Analysis

Let us take Eq. (15) and define the quantity

$$m_p = (\tau_0/2)m \tag{16}$$

as the photometric mass. It has the units 0 mag sec^4/cm^3. If the final mass is negligible, then

$$m_p = \int_t^{t_e} \frac{I}{v^3}\, dt' \tag{17}$$

This assumption is valid for those faint meteors which do not penetrate to low altitudes. Combining Eqs. (5) and (16), we define the mass parameter K as

$$K = \frac{\rho v^2}{m_p^{1/3}\,(-dv/dt)} \tag{18}$$

or

$$K = \frac{4m_p^{2/3}}{\tau_0\, C_D A} \equiv \frac{2^{4/3}\,\rho_B^{2/3}}{\tau_0^{1/3}\, C_D A^*} \tag{19}$$

where the reference area A^* is related to the cross-sectional area by

$$A = A^* m^{2/3} \rho_B^{-2/3} \tag{20}$$

(For example, $A^* = 1.21$ for a sphere and 1.50 for a cube.) Writing the dynamic equation in this form is convenient, since K will be constant (if τ_0 and C_D are constant) as long as the body maintains the same shape. This is probably a better assumption than assuming the coefficient ϕ to be constant, especially if considerable mass is lost during entry. Since all quantities on the right-hand side of Eq. (18) are known, K can be determined readily from Eqs. (17) and (18). If τ_0 is known and an assumption is made as to the shape of the body, the meteor density is given by Eq. (19). The mass, of course, can be obtained independent of K from Eq. (17).

Some experimentally determined values of K are shown in Table 2. Whipple[30] used recent values of τ_0 to obtain meteor densities of 0.4 g/cm^3. This value may not correspond to any particular composition but rather illustrates the mean of densities associated with photographic meteors. It should be pointed out that not all cometary meteors have the low value of K given in Table 2; Cook et al.[27] analyzed two such meteors, one of which yielded $\log K = 6.79$. This meteor was slow, however, and penetrated to 40 km, and probably was not in free-molecule flow during all of its luminous flight.

A refinement to this technique is often used when, as in the preceding case, the end mass is not negligible. Then Eq. (17) is replaced by

$$m_p - m_{p_e} = \int_t^{t_e} \frac{I}{v^3}\, dt' \tag{21}$$

Equations (6) and (15) can be combined to eliminate the rate of change of the mass, yielding

$$I/\rho v^6 m_p^{2/3} = \sigma/K \tag{22}$$

where

$$\sigma = C_H/C_D Q \tag{23}$$

and where K is given by Eq. (19). The mass m_p and the parameters σ and K are all functions of time. The system to be solved is underspecified, since there are only three equations and four unknowns (m_p, m_{pe}, σ, and K). Although it is not clear from their paper, Millman and Cook[11] evidently assumed that σ/K was constant toward the end of the trajectory, whence Eqs. (21) and (22) give σ and K.¶ If the ratio σ/K is not constant, then some other procedure [perhaps assuming K is constant in Eq. (18)] must be followed, such as in Ref. 27. Either approach is, in the writer's opinion, less satisfactory than one in which the complete system, including Eq. (23), is used to determine the mass. Of course, this implies that C_H, C_D, and Q must be known functions of m, ρ, v, and the meteoroid composition. Before discussing some recent papers in which this is done, two more methods for estimating mass will be mentioned. These are the dynamical and kinetic approaches to the problem.

Dynamical Approach

In free-molecule flow, the coefficients C_H and C_D are assumed to be constant. In this case, we can divide Eq. (5) by Eq. (6) and integrate from entry conditions to get

$$m/m_\infty = \exp[-(\sigma/2)(v_\infty^2 - v^2)] \tag{24}$$

where σ is given by Eq. (23). This technique has been used by Gazley,[31] by Fesenkov[32, 33] in his analysis of the entry of the great Sikhote-Alin meteorite, by Ceplecha[19] for the Přibram meteorite, and by Bronshten[34, 35] in his work on Tunguska and Kaalijarv. But it is clear that the mass variation obtained from Eq. (24) is not realistic for objects that penetrate to low altitudes; this fact is so stated by Ceplecha[19] and Bronshten.[53]

The coefficient σ, which enters into both Eqs. (24) and (6) in the form σC_D, has been measured in connection with several meteor programs to find a mass-luminosity law for the distribution of meteors or to determine the atmospheric density at high altitudes. Jacchia[36] showed that σ depended on the integrated brightness of the meteor and attributed the velocity dependence that σ showed to flaring or fragmentation. The reason for this is evident if Eqs. (5) and (6) are written in differential form. Then

$$\sigma = \frac{(1/m)(dm/dt)}{v(dv/dt)} \tag{25}$$

The numerator is a photometric quantity

$$\frac{1}{m}\frac{dm}{dt} = \frac{I/\tau v^2}{m_e + \int (I/\tau v^2)\, dt} \tag{26}$$

If m_e is negligible, if the luminous efficiency coefficient is constant over the trajectory, and if an average velocity is used, then

$$\frac{1}{m}\frac{dm}{dt} \approx \frac{I}{\int I dt} = \frac{I}{E} \tag{27}$$

which is the form used by Jacchia.[36] The total brightness is a function of all of the objects that make up the meteor luminosity; the dynamic motion, obtained from rotating shutter cameras, is a function primarily of the largest pieces. Hence σ essentially reflects the measurement technique and will vary with it. In his book, Levin[5] tried to show that σ varied because of aerodynamic reasons, as one would intuitively expect because of shielding effects on the heat-transfer coefficient. The correlation (assuming that $C_H \sim \rho v R_B$) was not successful. Later,[37] he showed from the data of Ref. 17 that the luminous path length decreased as the maximum luminous intensity increased, and that Eq. (27) was not valid for the mass-luminosity variation. The data also indicated that most meteors fragment.

The effect of fragmentation in an aerodynamic sense on C_H/C_DQ has not been determined. However, Gazley's value of σ, estimated for selected photographic meteors from the variation of velocity with altitude up to peak luminosity, corresponded to an accommodation coefficient of about one. The values quoted by Jacchia, tabulated also in Table 3, correspond to an accommodation coefficient 10 times higher. It is evident that σ does not represent the actual aerodynamic variation of C_H/C_DQ but also includes energy contributions due to crumbling, fragmentation, and perhaps air radiation. For this reason, the values of σ given in Table 3 are most useful in providing order-of-magnitude estimates for the mass variation.

It is also of interest to show the velocity law when the mass is variable. Inserting Eq. (24) into Eq. (5) and integrating, we obtain

$$\rho\left[\frac{C_DA}{2m_\infty\alpha\ \sin\gamma}\right] = \frac{\rho}{\phi} = \frac{1}{2\exp[(\sigma/2)\, v_\infty^2]} \times \left[\overline{Ei}\left(\frac{\sigma}{2}v_\infty^2\right) - \overline{Ei}\left(\frac{\sigma}{2}v^2\right)\right] \tag{28}$$

where $\overline{Ei}$ is the exponential integral. Notice that it is necessary to assume the parameter ϕ constant in order to integrate; ϕ may be evaluated at entry, whereupon $\phi = \phi_\infty$, or we may assume that C_DA is constant at some other specified value. Equation (28) reduces to Eq. (8) only if $\sigma = 0$, although it is necessary to use a constant ballistic parameter for both solutions. In this sense, Eq. (28) is not really self-consistent, since ϕ will vary if $\sigma > 0$.

The velocity law where a constant K is used assumes a different form because the variable mass enters Eq. (18) only to the $\frac{1}{3}$ power. Integration here yields

$$\frac{\rho}{\alpha\, K(m_{p\infty})^{1/3}\sin\gamma} = \frac{1}{2\exp(\sigma v_\infty^2/6)} \times \left[\overline{Ei}\left(\frac{\sigma}{6}v_\infty^2\right) - \overline{Ei}\left(\frac{\sigma}{6}v^2\right)\right] \tag{29}$$

By virtue of its definition [Eq. (19)], K is independent of any change in dimension as long as the shape (or C_DA^*) remains constant. Therefore, Eq. (29) is more self-consistent than Eq. (28), but neither is valid when σ varies greatly.

Table 3 Values of entry parameter σ

Source	Average velocity, km/sec	σ, sec^2/cm^2	Comments
Levin[5]:	...	2×10^{-12}	Strong shielding
	...	6.3×10^{-12}	Weak shielding
Jacchia[36]:	17	9.14×10^{-12}	Based on Super-Schmidt measurements
	22.5	7.78×10^{-12}	
	27.3	6.61×10^{-12}	
	31.8	5.76×10^{-12}	
	36.7	6.04×10^{-12}	
	44.3	5.14×10^{-12}	
	56.3	4.61×10^{-12}	
	65.3	4.69×10^{-12}	
Bronshten[29]:	7	0.2×10^{-12}	
	10	0.4×10^{-12}	
	14	0.6×10^{-12}	
	17	0.8×10^{-12}	
	20	0.9×10^{-12}	
Bronshten[29]:	...	10^{-12}	
Gazley[31]:	...	0.4×10^{-12}	Estimated at peak luminosity of photographic meteors

¶ Contrary to the statement in Ref. 11, a plot of $(\sigma/K)^{3/2}m_p$ vs $m_p - m_{pe}$ does not determine both σ/K and m_{pe}; it will only yield σ/K vs m_{pe}.

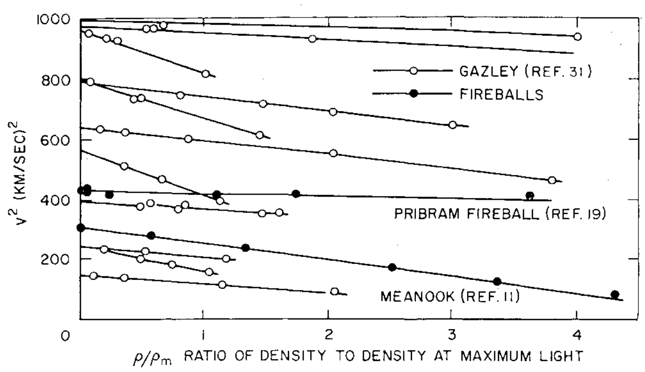

Fig. 3 Specific kinetic energy loss v^2 during entry as a function of density ratio.

Kinetic Approach

For meteors that do not decelerate appreciably during luminous flight, it can be assumed that the specific kinetic energy loss during flight is determined by the initial value. This technique was suggested by Gazley.[31] From Eq. (5)

$$(d/dt)(v^2) \approx -(C_DA/m)_\infty \rho v^3{}_\infty \quad (30)$$

and, from Eq. (7),

$$(d/d\rho)(v^2) = -(C_DA/m)_\infty \cdot (v_\infty{}^2/\alpha \sin\gamma) \quad (31)$$

Equation (31) can be integrated to yield

$$v^2/v_\infty{}^2 = 1 - (2v_\infty{}^2/\phi_\infty)\rho \quad (32)$$

Equation (32) is a linear equation in known functions v^2 and ρ with unknown coefficients $v_\infty{}^2$ and ϕ_∞. These can be determined by curve-fitting the measured values. Figure 3 shows examples of this procedure for obtaining the initial mass and velocity. Most of the examples were taken from Gazley.[31] Two recent fireballs also are shown in Fig. 3. The linear behavior of v^2 and ρ is followed in both cases until the bodies decelerate greatly. Although it does not show on this scale, the Meanook meteor was already decelerating when the first measurements were made. This method appeared to be quite successful in predicting the mass of the Přibram meteorite, as discussed later.

New Techniques

Undoubtedly, some of the objects analyzed by the previous techniques are experiencing continuum flow. This is certainly true in the case of the fireballs. One can therefore question the validity of the photometric approach in which τ_0 is obtained from meteor measurements, or the dynamic approach in which σ is assumed to be constant and is also obtained by photometric methods.

There are essentially two solutions to the problem of finding the mass under these conditions. The first consists of an extrapolation of known hypersonic heat-transfer laws to higher velocities. An empirical luminous-efficiency equation may or may not be used here. The second technique consists of predicting new heat-transfer equations for the higher velocities.

Cook, Jacchia, and McCrosky[27] carried out an extensive analysis of the heating, ablation, and vaporization of iron meteors. Using constant-density stagnation heat transfer to a sphere in the form[38]

$$C_H = 2.26/(Re_s)^{1/2} \sim (1.6\ \lambda/R_B)^{1/2} \quad (33)$$

and the response to such heating from a melting or vaporizing iron sphere, they estimated bounds on the radius for both a rotating and nonrotating body. The same analysis using free-molecule flow gave upper bounds for the radius when the height corresponding to the onset of ablation was specified. Somewhat the same technique was employed by Cook[39] for a stony oblate spheroid rather than an iron sphere. The meteoric stone was assumed to ablate in a glassy manner, similar to tektites. Cook inferred from this work that a recent slow meteor which penetrated to 48 km passed from ablation by melting and spraying to vaporization and back to melting during its luminous flight. The onset of evaporation in free-molecule flow has also been studied recently by Ceplecha and Padevet.[40]

An analysis similar to Cook's was independently performed by Chapman and Larson[41] in their work on the origin of tektites. However, more realistic values for the stagnation heat transfer were used than those in Refs. 27 and 39. That is,

$$C_H = \frac{q}{\frac{1}{2}\rho v^3} = \frac{2220 v^{0.15}}{(\rho R_B)^{1/2}}\left(1 - \frac{H_B}{H_s}\right) \quad (34)$$

This equation is well established by both analytical solutions of the full boundary-layer equations[42] and experimental correlations in shock tubes up to 10 km/sec.[43] The equation does not, however, account for ionization or radiation effects. (These are not important for small, slow objects like tektites). Chapman also utilized the experimentally determined formula[44]

$$C_{H_{\text{trans}}} = C_{HFM}C_H/(C_{HFM}{}^2 + C_H{}^2)^{1/2} \quad (35)$$

for the transition zone heat-transfer coefficient, where C_H is given by Eq. (34) and C_{HFM} is the free-molecule value at any Reynolds number.

Riddell and Winkler[45] evaluated the heat transfer to bodies with low values of m/C_DA. These objects lose all of their kinetic energy before impacting the earth. Radiation losses were taken into account when the equilibrium stagnation-point conditions were computed. The authors showed, under the assumption that mass losses were small, that 20% of the available kinetic energy was absorbed by a body entering at 42.6 km/sec. Since this conclusion invalidated the assumption of negligible mass loss, they performed an analysis of the entry when the mass varied. A constant value of $\sigma = 8.34 \times 10^{-12}$ was used, based on measurements of meteor luminosity and deceleration reported by Whipple.[46] Their investigation for velocities from 18.3 to 42.6 km/sec showed that objects in the range 0.5 cm to 50 m would experience mass loss in excess of 20%.

Fay, Moffatt, and Probstein[20] also studied the problem of meteorite survival. It was assumed that the ballistic parameter ϕ was constant at a value determined by the altitude of peak heating. The total ablated mass was found as a function of the integrated heat input. The assumption that ϕ = const implied that the mass loss due to ablation be a small percentage of the mass at peak heating in order for the analysis to be self-consistent. Radiative effects were not considered in the shock calculations but were included in the heat transfer.

They determined the stagnation-point convective heat transfer for a multiply ionized gas in thermodynamic equilibrium. The boundary-layer composition was assumed to be constant (deionization did not occur at the wall). The Newtonian velocity gradient was used, and the temperature gradient at the wall was found by assuming that the thermal boundary layer was large. This is a result of the high electronic thermal conductivity or low Prandtl number. A higher heat transfer would result if recombination and deionization occurred at the wall. The radiative heat transfer was calculated for the extremes of a transparent and an opaque gas. Continuum radiation was assumed to occur (it was found that line radiation heating exceeded continuum heating only for small bodies in which convective heating was dominant).

Their results indicated that the convective and radiative heat-transfer coefficients were nearly independent of velocity for $20 \le v \le 70$ km/sec; radiative heating became important at altitudes below 25 km when the shock layer became optically thin. The mass-loss analysis showed that a nonvaporizing body will not survive ($\Delta m/m > 0.5$) unless it is large enough to experience peak heating above sea level (for a vertical entry, the radius must be larger than 10 m). A minimum occurred in the mass loss as a function of radius when convection and transparent radiation heating were equal. For vaporizing bodies, survival is insured if the radius lies between a fraction of a centimeter and several centimeters. If the ablating material is opaque, this size range is somewhat larger. The entry of very small bodies was not considered in Ref. 20.

Bronshten also discussed the problems encountered during the entry of large bodies. His book[29] presents a comprehensive survey of the literature, both Soviet and Western, on the temperature and density in the shock layer, the effect of radiation on the shock structure, relaxation effects, and heat transfer. The latter includes both radiative and electronic contributions; the usual convective heat transfer is based on the work of Fay and Riddell[42] for a dissociated gas.

V. Examples

To illustrate the various theories discussed in Sec. IV, we have selected two recent fireballs, Meanook and Přibram, which were, fortunately, photographed during entry. These are treated only briefly, since the Meanook firebll has been summarized well in Refs. 11 and 47, and the Přibram fireball has been discussed at length in Ref. 19.

Meanook

The Meanook fireball was observed photographically in 1952 at the Meanook (Canada) Meteor Observatory. No associated meteorite was found. Millman and Cook[11] discussed the reduction of data and the spectrum and presented values for the mass variation and the coefficients K and σ based on the data and on Jacchia's[24] value of τ_0 ($\log\tau_0 = -18.19$). The peak zero-magnitude luminous intensity was over 1000, corresponding to a maximum zenith magnitude of about -7.5. The methods used to derive the mass were based on the photometric technique discussed in Sec. IV.

Allen and Yoshikawa[47] took the data of Ref. 11 and obtained values for the mass variation under the assumption that the object was in continuum flow. A portion of the luminosity was assumed to be due to gas cap radiation, although the analysis shows (again using Jacchia's value of τ_0) that the difference between total radiation and gas cap radiation is small. However, it is possible that the authors underestimated the contribution from the gas cap, since they assumed the air was in equilibrium. Further, the observed luminosity was obtained from measurements in the spectral range from 0.36 to 0.66 μ, whereas the actual radiation from the shock at high velocities contains important contributions in the ultraviolet. This is especially true if the air is not in equilibrium.[22] It is not presently known how much light this nonequilibrium radiation would produce, or whether the emitted light would be more in the ultraviolet or the visible. If it were in the ultraviolet, it would essentially be "lost," since it would be absorbed before reaching the spectrograph. Even so, it would still form a part of the kinetic energy lost by the meteoroid and should be taken into account. Therefore, the question of the applicability of Eq. (11) and its associated value of τ_0 must remain unanswered until further experiments on the radiation from strong shocks are performed.

Although Allen and Yoshikawa did not compare their results for the mass variation of Meanook with those of Millman and Cook, we thought such a comparison would illustrate quite well the differences between the various theories. Allen and Yoshikawa assumed the body was a sphere and deduced (on the basis of the luminosity) that the density was about that of water or less. Figure 4 was prepared from these facts and the variation of $\rho_B R_B$ given in Ref. 47. Jacchia's value of τ_0 was used to convert the photometric values of Ref. 11 into actual mass. In addition, we calculated m/m_∞ from Eq. (24), using a value of $\sigma = 2.22 \times 10^{-12}$ (the mean of the measured values of Ref. 11 during the early part of the trajectory) and also the value 10^{-12} from Bronshten.[29] (See Fig. 2 for the velocity variation of the Meanook meteor.) In these calculations we assumed that $m_\infty = 1270$ g, the value found in Ref. 11. Allen and Yoshikawa[47] did not give a value for m_∞.

Figure 4 shows that the variation of mass with altitude is quite different for the photometric and continuum models, although the two curves appear to converge at lower altitudes. (Actually, an extrapolation of the curve of Ref. 47 to lower altitudes yields a final mass that is a factor of 10 higher for that case). The original mass at entry appears to be higher for the photometric analysis, but it is difficult to verify this, since the body was already decelerating and losing mass before the fireball came into view of the meteor camera. It is evident from the two curves of Fig. 4 that the parameter σ is probably not constant; Ref. 11 indicates that σ varied from 1.87×10^{-12} to 8×10^{-12} as the object entered.

The dynamical method of Gazley[31] was used to estimate the original mass. Since the body was already decelerating, it was necessary to calculate ϕ from the deceleration and to extrapolate to the entry conditions ($\rho \to 0$, $v \to v_\infty$, $dv/dt \to 0$). On this basis, with the further assumption that the body was a sphere of density $\rho_B = 1$ g/cm^3, we estimated $m_\infty = 1096$ g, which is in rough agreement with the trend of both curves.

We also see from Fig. 4 that if low values of σ are used in Eq. (24) one might overestimate the mass at the end of heating. This would be a serious error if one were searching for an associated meteorite. (It is also possible to overestimate the value of the original mass by this method).[19]

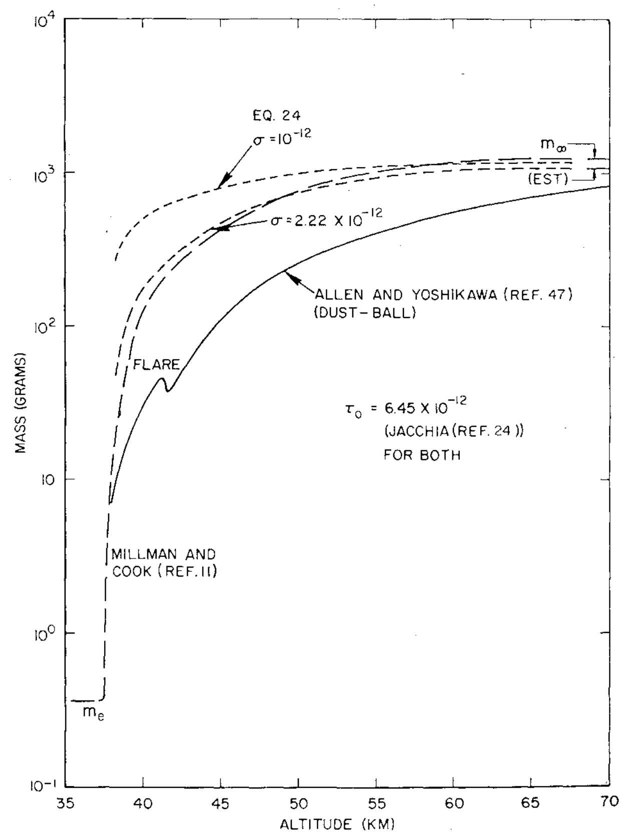

Fig. 4 Variation of mass with altitude during photographed flight of the Meanook fireball.

Přibram

The fireball that passed over Czechoslovakia on April 7, 1959, is the only other case where such an event was photographed with detail sufficient for the determination of a trajectory. Furthermore, four specimens of the original meteorite were recovered with a combined mass of 5.8 kg. This provided a unique opportunity for the simultaneous study of the structure of the meteorite and also its behavior in the atmosphere.

The Přibram fireball "entered" when it became visible as an object of -0.6^m (absolute) on the plates of meteor cameras at stations Ondřejov and Přcice, 40.388 km apart. The Ondřejov cameras were equipped with rotating shutters, and the velocity and deceleration could, in theory, be determined. Unfortunately the images were badly overexposed, and so there is an extreme amount of scatter in the data. It was not possible to obtain realistic values of the deceleration.

The entry was determined by Ceplecha[19] to occur at 97.8 km, with an entry velocity of 20.886 km/sec. The inclination of the trajectory seen from Ondřejov (where the object passed almost overhead) was estimated to be 43° to the horizontal. The intensity of light from the fireball was brilliant; it rose almost exponentially to an estimated -19 magnitude until an altitude of 54 km was reached. At this point the body apparently began to break up, since at 44 km a piece was observed to separate. Eventually 17 individual trails were recorded.

A comprehensive study of the entry, the dispersion, and the recovered pieces of the Přibram meteorite is presently being conducted by the Astronomical Institute of the Czechoslovak Academy of Sciences.[19, 48–50] The methods utilized in their study of the entry and their estimates of the original size of the body are, however, based partially on the formulas developed for the entry of small meteoroids less than 1 cm in diameter.[5, 16]

To obtain the initial velocity and mass, Ceplecha used an approximate solution of Eq. (5) in a least-squares curve fit to the photographic data. Several values for the initial mass are given in Table 4 for three assumed values of the drag coefficient. We were, however, unable to verify these values of m_∞ using the numbers cited in Ref. 19 for the curve fit.

Because of this, we decided to obtain an estimate of the mass using the kinetic technique of Eq. (32), which gave such good agreement in the case of the Meanook fireball. This value, $m_\infty = 68$ kg for $C_D = 1$, appears to be about a factor of 10 lower than that obtained by Ceplecha for the same value of C_D.

Another estimate of m_∞ can be obtained from Eq. (8), which is the solution of the dynamic equation for a constant ballistic coefficient. (This method is nearly equivalent to the technique employed by Ceplecha, except that he let the scale height α be one of the undetermined constants in the curve-fit.) These results are shown in Table 4, where it is evident that the present estimates of mass are about a factor of 10 lower than that given in Ref. 19 for $C_D = 1$.

Table 4 Estimates of the initial mass of the Přibram meteorite

Source	Initial mass, kg	Comments
Ceplecha[19]:	700	$C_D = 1$
	2000	$C_D = 1.4$
	5000	$C_D = 2.0$
Eq. (32):	68	Six-point least-squares fit, spherical shape, $C_D = 1$
Eq. (8):	60	Six-point weighted least-squares curve-fit using data of Ref. 19, spherical shape, $C_D = 1$

Ceplecha also estimated values for the mass at the end of luminous flight which indicated that 90% or more of the original mass was lost during luminous flight. This should occur only if the body experiences free-molecule heating over the luminous trajectory, or is smaller in size. According to the curves of Ref. 20, a stone sphere weighing 700 kg would not lose 90% of its mass even if the entry were vertical. On this basis we would expect that more of the meteorite should have survived. If, however, the initial mass were, say, 60–100 kg, then the curves of Ref. 20 indicate that nearly all the original mass survived early heating, which would be in agreement with Ceplecha's estimates that the combined mass at the end of the luminous flight was about 70 kg.

Both of these analyses indicate that there may remain several large pieces which have not yet been found. The Astronomical Institute of the Czechoslovak Academy of Sciences is continuing the search.

VI. Conclusions

This survey has indicated that contemporary analyses of meteor entry take essentially two different forms. The success of the empirically based approach of the Smithsonian group, illustrated by the Meanook study,[11] depends on an accurate value for the efficiency with which kinetic energy is transformed into light over a given spectral range. Studies in recent years, summarized in Ref. 21, show that the luminous efficiency factor can vary by more than two orders of magnitude. Some of this is due to differences in meteoroid composition and shape, but some is undoubtedly due to fragmentation and gas radiation. For example, nonfragmenting meteors apparently produce light more efficiently than do those that crumble. Fireballs, on the other hand, would have to be twice as dense as meteors in order to have the same luminous efficiency.[21]

The other method of solving the entry equations makes certain assumptions about the variation of the aerodynamic coefficients during entry. This approach was used extensively in Europe and the Soviet Union. Because it is necessary to know the size of the body in order to use the correct drag and heat transfer, this technique also has limitations. The masses and densities calculated on this basis are not likely to agree with those calculated using the photometric method; the former are for an integral body, whereas the latter are derived from bodies that may or may not fragment in flight.

It is natural to ask which of these methods holds more promise for the analysis of the entry of larger objects. For many reasons, it is unlikely that an empirical approach will be successful. The primary objections to the use of a luminous efficiency for the deduction of mass are the following. If the gas cap radiation does contribute to the light, the intensity and spectral distribution will be a function of the velocity, ambient density, and radiating volume, which is proportional to the body size. If the gas cap radiation does not contribute to the measured light (perhaps because it is in the ultraviolet) it still contributes to the heat transfer, which determines the luminous ablated mass. What is measured, then, will still be a function of body size, ambient density, and velocity (as well as composition).[52] Furthermore, the radiation from ablated mass in continuum flow can be of different structure than that which takes place near a vaporizing meteor where the ablated mass encounters a high-energy airstream (see, e.g., Ref. 12).

Although we feel that the aerodynamic approach (Refs. 20, 29, 41, 45, and 47) is more promising, many outstanding problems remain before such methods can be used with confidence. The most significant problem is directly connected with the radiation during entry. It is still not clear how objects that are suspected to be as small as the Přibram meteorite, or Treysa,[51] or even Meanook (which was never found), can produce what both the eye and camera would interpret as an exceedingly large, brilliant, pear-shaped ball of fire. This radiation cannot come from the body itself, since the melting temperatures are quite low. It is only after peak heating and deceleration occur that the body radiation would be evident. If, as suspected, the gas cap radiation does not peak in the visible regions of the spectrum, the light must be due to ablated mass. Under normal circumstances this mass is confined to the viscous flow regimes. One would expect therefore that the largest amount of this radiation would be from the wake and boundary layer and not from regions far from the body (see, e.g., Ref. 52).

Thus, if the "fireball" effect is not an optical phenomenon, it is difficult to explain on the basis of our present knowledge of gasdynamics unless the ultraviolet radiation from the shock produces fluorescence of the ambient air. This is, however, pure conjecture, since the nature of gas cap radiation at velocities in excess of 10 km/sec has not yet been experimentally studied. Additional work is also necessary in the area of radiation-coupled flow fields and heat transfer in ionized gases before the aerodynamic approach to the entry problem rests on a firm structure.

For the entry of small objects, such as those that produce the photographic meteors of Fig. 2, use of the empirical luminous efficiency technique appears most promising, especially if controlled experiments such as those of Ref. 25 are continued.

References

[1] Öpik, E. J., *Physics of Meteor Flight in the Atmosphere* (Interscience Publishers, Inc., New York, 1958).

[2] Greenhow, J. S. and Hawkins, G. S., "Ionizing and luminous efficiencies of meteors," Nature **170,** 355–356 (August 1952).

[3] *U. S. Standard Atmosphere, 1962* (U. S. Government Printing Office, Washington, D. C., December 1962).

[4] Baker, R. M. L., Jr. and Charwat, A. F., "Transitional correction to the drag of a sphere in free-molecule flow," Phys. Fluids **1,** 73–81 (1958).

[5] Levin, B. Iu., "The physical theory of meteors, and meteoric matter in the solar system," Chaps. I–III, transl. by J. Miller and D. Kraus, ASTIA Doc. AD-110091; transl. of *Fizicheskaia Teoriia Meteorov i Meteornoe Veshchestvo v Solnechnoi Sisteme* (Izd-vo, Moscow, 1956), Chaps. 1–3.

[6] Hayes, W. D. and Probstein, R. F., *Hypersonic Flow Theory* (Academic Press, New York, 1959), 1st ed., Chap. X.

[7] Van Dyke, M., "Second order compressible boundary layer theory with application to blunt bodies in hypersonic flow," Stanford Univ., Dept. of Aeronautical Engineering, Rept. 112 (July 1961).

[8] Willett, J. E. and Lehto, D. L., "Normal shock (Rankine-Hugoniot) relations for various altitudes from sea level to 300,000 feet," U. S. Naval Ordnance Test Station NAVORD Rept. 6075 (April 1958).

[9] Cook, A. F. and Hawkins, G. S., "The meteoric head echo," Smithsonian Contrib. Astrophys. **5,** 1–7 (1960).

[10] Kramer, E. N., Liubarsky, K. A., and Ivanikov, V. I., "Spectrographic observations of meteors in the USSR in 1957–1960," Smithsonian Contrib. Astrophys. **7,** 157–161 (1963).

[11] Millman, P. M. and Cook, A. F., "Photometric analysis of a spectrogram of a very slow meteor," Astrophys. J. **130,** 648–662 (1959).

[12] Astapovich, I. S. and Fedynskiy, V. V., "Progress in meteor astronomy in 1958–1961," JPRS transl. 23,047 (February 1964); transl. from Meteoritika **23,** (1963).

[13] Russell, J. A., "Molecular nitrogen in the spectra of two Perseid meteors," Publ. Astron. Soc. Pacific **72,** 291–295 (1960).

[14] Cook, A. F., "The nature of meteoric radiation," J. Atmospheric Terrest. Phys., Special Suppl. **2,** 8–15 (1955).

[15] Millman, P. M., "A general survey of meteor spectra," Smithsonian Contrib. Astrophys. **7,** 119–129 (1963).

[16] Whipple, F. L. and Hawkins, G. S., "Meteors," *Handbuch der Physik* (Springer-Verlag, Berlin, 1958), Vol. 52, pp. 519–564.

[17] Hawkins, G. S. and Southworth, R. B., "The statistics of meteors in the earth's atmosphere," Smithsonian Contrib. Astrophys. **2,** 349–364 (1958).

[18] Olivier, C. P., "Long enduring meteor trains and fireball orbits," Proc. Am. Phil. Soc. **101,** 296–315 (1957).

[19] Ceplecha, Zd., "Multiple fall of Příbram meteorites photographed," Bull. Astron. Soc. Czechoslovakia **12,** 21–47 (1961).

[20] Fay, J. A., Moffatt, W. C., and Probstein, R. F., "An analytical study of meteor entry," AIAA J. **2,** 845–855 (1964).

[21] Verniani, F., "On the luminous efficiency of meteors," Smithsonian Institution Astrophysical Observatory, Research and Space Science Special Rept. 145 (February 17, 1964).

[22] Allen, R. A., Rose, P. H., and Camm, J. C., "Nonequilibrium and equilibrium radiation at super-satellite re-entry velocities," Avco-Everett Research Rept. 156 (September 1962).

[23] Hundley, R. O., "Air radiation from nonequilibrium wakes of blunt bodies," The Rand Corp., RM-4071-ARPA (March 1964).

[24] Jacchia, L. G., "Ballistics of the upper atmosphere," Harvard College Observatory and Center of Analysis, Massachusetts Institute of Technology Rept. 2 (1948).

[25] McCrosky, R. E. and Soberman, R. K., "Results from an artificial iron meteoroid at 10 km/sec," Smithsonian Contrib. Astrophys. **7,** 199–209 (1963).

[26] Staniukovich, K. P., "Ob odnom effekte v oblasti aerodinamiki meteorov" ("Concerning a certain effect in the field of meteor aerodynamics"), AN SSSR, Otd. Tekhn. Nauk. Izv.: Mekh. i Mashinostr. **5,** 3–8 (1960), transl. by J. B. Gazley in Rand RM-2932-PR (January 1962).

[27] Cook, A. F., Jacchia, L. G., and McCrosky, R. E., "Luminous efficiency of iron and stone asteroidal meteors," Smithsonian Contrib. Astrophys. **7,** 209–221 (1963).

[28] Astapovich, I. S., *Meteornye Iavleniia v Atmosfere Zemli* (*Meteoric Phenomena in the Earth's Atmosphere*) (Fizmatgiz, Moscow, 1958).

[29] Bronshten, V. A., *Problemy Dvizheniia v Atmosfere Krupnykh Meteoritnykh Tel* (*Problems of the Motion of Large Meteoritic Bodies in the Atmosphere*) (Izd-vo, Moscow, 1963); also transl. by J. B. Gazley and M. F. Romig, Rand RM-4257-PR (to be published).

[30] Whipple, F. L., "On meteoroids and penetration," J. Geophys. Res. **68,** 4929–4939 (1963).

[31] Gazley, C., Jr., "Meteoric interaction with the atmosphere; theory of drag and heating and comparison with observations," *Aerodynamics of the Upper Atmosphere,* The Rand Corp., R-339 (1959).

[32] Fesenkov, V. G., "O dvizhenii Sikhote-Alinskogo meteorita v atmosfere" ("On the motion of the Sikhote-Alin meteorite in the atmosphere,") Meteoritika **9,** 3–26, 27–31 (1951).

[33] Fesenkov, V. G., "Zamechanii a o dvizhenii Sikhote-Alinskogo meteorite v atmosfere" ("Remarks on the motion of the Sikhote-Alin meteorite in the atmosphere,") Meteoritika **12,** 72–74 (1955).

[34] Bronshten, V. A., "K voprosu o dvizhenii v atmosfere Tungusskogo meteorita" ("Toward the question of the motion of the Tunguska meteorite in the atmosphere,") Meteoritika **20,** 72–86 (1961); also Staniukovich, K. P. and Bronshten, V. A., "O skorosti i energii Tungusskogo meteorita" ("On the velocity and momentum of the Tunguska meteorite,") Dokl. AN SSSR **140,** 583–586 (1961).

[35] Bronshten, V. A., "Ob obstoiatel'stvakh padeniia Kaaliiarvskogo meteorita" ("On the Circumstances of the descent of the Kaaliarv meteorite,") Meteoritika **22,** 42–46 (1962).

[36] Jacchia, L. G., "On two parameters used in the physical theory of meteors," Smithsonian Contrib. Astrophys. **2,** 181–187 (1958).

[37] Levin, B. Iu., "Fragmentation of meteoric bodies," Soviet Astron.—AJ **7,** 233–238 (1963).

[38] Probstein, R. F. and Kemp, N. H., "Viscous aerodynamic characteristics in hypersonic rarefied gas flow," J. Aerospace Sci. **27,** 174–193 (1960).

[39] Cook, A. F., "A criterion for the mode of ablation in stone meteors," Smithsonian Contrib. Astrophys. **4,** 131–136 (1963).

[40] Ceplecha, Zd. and Padevet, V., "The beginning of rapid evaporation of meteors of different dimensions," Bull. Astron. Soc. Czechoslovakia **12,** 191–195 (1961).

[41] Chapman, D. R. and Larson, H. K., "The lunar origin of tektites," NASA TN D-1556 (February 1963).

[42] Fay, J. A. and Riddell, F. R., "Theory of stagnation point heat transfer in dissociated air," J. Aeronaut. Sci. **25,** 73–85, 121 (1958).

[43] Detra, R. W., Kemp, N. H., and Riddell, F. R., "Addendum to heat transfer to satellite vehicles reentering the atmosphere," Jet Propulsion **27**, 1256–1257 (1957).

[44] Ferri, A. and Zakkay, V., "Measurements of stagnation point heat transfer at low Reynolds numbers," J. Aerospace Sci. **29**, 847–850 (1962).

[45] Riddell, F. R. and Winkler, H. B., "Meteorites and re-entry of space vehicles at meteor velocities," ARS J. **32**, 1523–1530 (1962).

[46] Whipple, F. L., "Meteors and the earth's upper atmosphere," Rev. Mod. Phys. **15**, 246–264 (1943).

[47] Allen, H. J. and Yoshikawa, K. K., "Luminosity from large meteoric bodies," Smithsonian Contrib. Astrophys. **7**, 181–195 (1963).

[48] Ceplecha, Zd., "Note on the mass-determination of the Příbram meteorites," Bull. Astron. Soc. Czechoslovakia **11**, 164 (1960).

[49] Ceplecha, Zd., "Experimental data on the final mass of the body landed on the earth after penetrating the atmosphere at cosmical velocity," Bull. Astron. Soc. Czechoslovakia **11**, 9–13 (1960).

[50] Vlasvek, J., "Multiple fall of Příbram meteorites photographed VI. Some results of aerodynamic measurements," Bull. Astron. Soc. Czechoslovakia **14**, 222–225 (1963).

[51] Heide, F., *Meteorites* (The University of Chicago Press, Chicago, Ill., 1964).

[52] Craig, R. A. and Davey, W. C., "Thermal radiation from ablation products injected into a hypersonic shock layer," NASA TN D-1978 (1963).

[53] Bronshten, V. A., private communication (January 1964).